Chapter Four

Discrete Fourier Transform

1-1 Introduction

The Discrete Fourier Transform (DFT) may be regarded as a logical extension of the Discrete-Time Fourier Transform (DTFT). By sampling the DTFT X (Ω) at uniformly spaced frequencies Ω = 2πk/N, where k = 0, 1, 2… N−1, we can define the DFT of x[n]. Let x[n], n = 0, 1, 2… N−1, be an N-point sequence. We define the DFT of x[n] as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}, \qquad k = 0, 1, 2, \ldots, N-1$$

The DFT can be represented by the operator "F" the opposite of it is the Inverse Discrete Fourier Transform (IDFT) denoted by F^{-1} as:

$$\boxed{\text{DFT}: \; X[k] = F[x[n]] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}}$$

$$\boxed{\text{IDFT}: \; x[n] = F^{-1}\left[X[k]\right] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi nk/N}}$$

Now for simplicity, we can use Matrices to calculate both DFT and IDFT. To do that we need to define the Twiddle factor "W" which is:

$W_N = e^{-j2\pi/N}$ now the equation of the DFT becomes:

$$X(k) = \sum_{n=0}^{N-1} x(n)\ W_N^{kn} \quad and\ IDFT\ is: \qquad x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\ W_N^{-kn}$$

Where n= 0, 1, 2, … , N-1

Prob. 1 : Obtain DFT of unit impulse $\delta(n)$.

Soln. :

Here $x(n) = \delta(n)$

According to the definition of DFT we have,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

But $\delta(n) = 1$ only at $n = 0$,

It is shown in Fig. F-2

Thus Equation becomes,

$$X(k) = \delta(0)\, e^{0} = 1$$

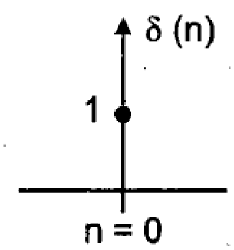


Fig. F-2

$$\boxed{\therefore\ \delta(n) \overset{DFT}{\longleftrightarrow} 1}$$

Prob. 2 : Obtain DFT of delayed unit impulse $\delta(n-n_0)$.

Soln. : We know that $\delta(n-n_0)$ indicates unit impulse delayed by 'n_0' samples.

Here $$x(n) = \delta(n-n_0) \quad \text{...(1)}$$

Now we have, $$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \quad \text{...(2)}$$

But $\delta(n-n_0) = 1$ only at $n = n_0$.

It is shown in Fig. F-3

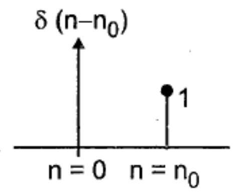


Fig. F-3

Thus Equation (2) becomes,

$$X(k) = 1 \cdot e^{-j2\pi kn_0/N}$$

$$\boxed{\therefore\ \delta(n-n_0) \overset{\text{DFT}}{\longleftrightarrow} e^{-j2\pi kn_0/N}}$$

Similarly we can write,

$$\boxed{\delta(n+n_0) \overset{\text{DFT}}{\longleftrightarrow} e^{j2\pi kn_0/N}}$$

Prob. 3 : Obtain N-point DFT of exponential sequence :

$$x(n) = a^n u(n) \text{ for } 0 \le n \le N-1$$

Soln. : According to the definition of DFT,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

Here $$x(n) = a^n u(n)$$

The multiplication of a^n with $u(n)$ indicates sequence is positive. Putting $x(n) = a^n$ in Equation we get,

$$X(k) = \sum_{n=0}^{N-1} a^n e^{-j2\pi kn/N}$$

$$\therefore \quad X(k) = \sum_{n=0}^{N-1} (ae^{-j2\pi k/N})^n$$

Now use the standard summation formula,

$$\sum_{k=N_1}^{N_2} A^k = \frac{A^{N_1} - A^{N_2+1}}{1-A}$$

Here $N_1 = 0$, $N_2 = N-1$ and $A = ae^{-j2\pi k/N}$

$$\therefore \quad X(k) = \frac{(ae^{-j2\pi k/N})^0 - (ae^{-j2\pi k/N})^{N-1+1}}{1-ae^{-j2\pi k/N}}$$

$$\therefore \quad X(k) = \frac{1-a^N e^{-j2\pi k}}{1-ae^{-j2\pi k/N}}$$

Using Euler's identity to the numerator term, we get,

$$e^{-j2\pi k} = \cos 2\pi k - j \sin 2\pi k$$

But k is an integer

$$\therefore \quad \cos 2\pi k = 1 \text{ and } \sin 2\pi k = 0$$

$$\therefore \quad e^{-j2\pi k} = 1 - j0 = 1$$

$$\therefore \quad X(k) = \frac{1-a^N}{1-a\,e^{-j2\pi k/N}}$$

$$\boxed{\therefore \; a^n u(n) \overset{DFT}{\longleftrightarrow} \frac{1-a^N}{1-a\,e^{-j2\pi k/N}}}$$

Prob. 4 : Find the DFT of following window function,

$$w(n) = u(n) - u(n-N)$$

Soln. : According to the definition of DFT,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \qquad ...(1)$$

The given equation is $x(n) = w(n) = 1$ for $0 \le n \le N-1$. We will assume some value of N. Let N = 4; so we will get 4-point DFT.

$$\therefore \quad X(k) = \sum_{n=0}^{3} 1 \cdot e^{-j2\pi kn/4} \qquad ...(2)$$

The range of k is from '0' to $N-1$. So in this case 'k' will vary from 0 to 3.

For $k = 0 \Rightarrow X(0) = \sum_{n=0}^{3} 1 \cdot e^{0} = \sum_{n=0}^{3} 1 = 1+1+1+1 = 4$

For $k = 1 \Rightarrow X(1) = \sum_{n=0}^{3} e^{-j2\pi n/4}$

$$\therefore \quad X(1) = e^{0} + e^{-j2\pi/4} + e^{-j4\pi/4} + e^{-j6\pi/4}$$

$$\therefore \quad X(1) = 1 + \left(\cos\frac{2\pi}{4} - j\sin\frac{2\pi}{4}\right) + \left(\cos\frac{4\pi}{4} - j\sin\frac{4\pi}{4}\right) + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right)$$

$$\therefore \quad X(1) = 1 + (0-j) + (-1-0) + (0+j)$$

$$\therefore \quad X(1) = 1 - j - 1 + j = 0$$

For $k = 2 \Rightarrow X(2) = \sum_{n=0}^{3} e^{-j2\pi \times 2n/4} = \sum_{n=0}^{3} e^{-j\pi n}$

$$\therefore \quad X(2) = e^{0} + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi}$$

$$\therefore \quad X(2) = 1 + (\cos\pi - j\sin\pi) + (\cos 2\pi - j\sin 2\pi) + (\cos 3\pi - j\sin 3\pi)$$

$$\therefore \quad X(2) = 1 + (-1-0) + (1-0) + (-1-0) = 1 - 1 + 1 - 1 = 0$$

For $k = 3 \Rightarrow X(3) = \sum_{n=0}^{3} e^{-j2\pi \times 3n/4} = \sum_{n=0}^{3} e^{-j6\pi n/4}$

$$\therefore \quad X(3) = e^{0} + e^{-j6\pi/4} + e^{-j3\pi} + e^{-j9\pi/2}$$

$$\therefore \quad X(3) = 1 + \left(\cos\frac{6\pi}{4} - j\sin\frac{6\pi}{4}\right) + (\cos 3\pi - j\sin 3\pi) + \left(\cos\frac{9\pi}{2} - j\sin\frac{9\pi}{2}\right)$$

$$\therefore \quad X(3) = 1 + (0+j) + (-1-0) + (0-j) = 1 + j - 1 - j = 0$$

$$\boxed{\therefore \quad X(k) = \{4, 0, 0, 0\}}$$

We can solve equations using W_N, let x(n) be written as x_N and X(k) written as X_k

$$x_N = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ \cdot \\ x(N-1) \end{bmatrix}_{N \times 1} \qquad X_k = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \cdot \\ \cdot \\ X(N-1) \end{bmatrix}_{N \times 1}$$

$$W_N^{kn} = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ \vdots \\ k=N-1 \end{array} \begin{array}{c} \begin{array}{ccccc} n=0 & n=1 & n=2 & \ldots\ldots & n=N-1 \end{array} \\ \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix}_{N \times N} \end{array}$$

Thus DFT can be represented in the matrix form as,

$$X_N = [W_N] x_N$$

Similarly, IDFT can be represented in the matrix form as,

$$x_N = \frac{1}{N}\left[W_N^* \right] X_N$$

Here W_N^* is complex conjugate of W_N.

We have, $W_N = e^{-\frac{j2\pi}{N}}$ $\therefore$ $W_N^{kn} = e^{-\frac{j2\pi}{N}\times kn}$

But N = 8 $\therefore$ $W_8^{kn} = e^{-\frac{j2\pi}{8}\times kn} = e^{-\frac{j\pi}{4}\times kn}$

Value of kn	$W_8^{kn} = e^{-j\frac{\pi}{4}\times kn}$	Value of the phasor
0	$W_8^0 = e^0$	1
1	$W_8^1 = e^{-j\frac{\pi}{4}\times 1} = e^{-j\frac{\pi}{4}}$	$0.707 - j\,0.707$
2	$W_8^2 = e^{-j\frac{\pi}{4}\times 2} = e^{-j\frac{\pi}{2}}$	$0 - j\,1$
3	$W_8^3 = e^{-j\frac{\pi}{4}\times 3} = e^{-j\frac{3\pi}{4}}$	$-0.707 - j\,0.707$
4	$W_8^4 = e^{-j\frac{\pi}{4}\times 4} = e^{-j\pi}$	-1
5	$W_8^5 = e^{-j\frac{\pi}{4}\times 5} = e^{-j\frac{5\pi}{4}}$	$-0.707 + j\,0.707$
6	$W_8^6 = e^{-j\frac{\pi}{4}\times 6} = e^{-j\frac{3\pi}{2}}$	$0 + j\,1$
7	$W_8^7 = e^{-j\frac{\pi}{4}\times 7} = e^{-j\frac{7\pi}{4}}$	$0.707 + j\,0.707$

And for 4-points (N=4) W_N is:

$$\begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

And for 2-points (N=2) W_N is:

$$W_2^{kn} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Solved problems using W_N (Twiddle factor):

Prob. 1 : Determine 2-point and 4-point DFT of a sequence,

$$x(n) = u(n) - u(n-2)$$

Sketch the magnitude of DFT in both the cases.

Soln. : First we will obtain the sequence $x(n)$. It is represented as shown in Fig. F-5.

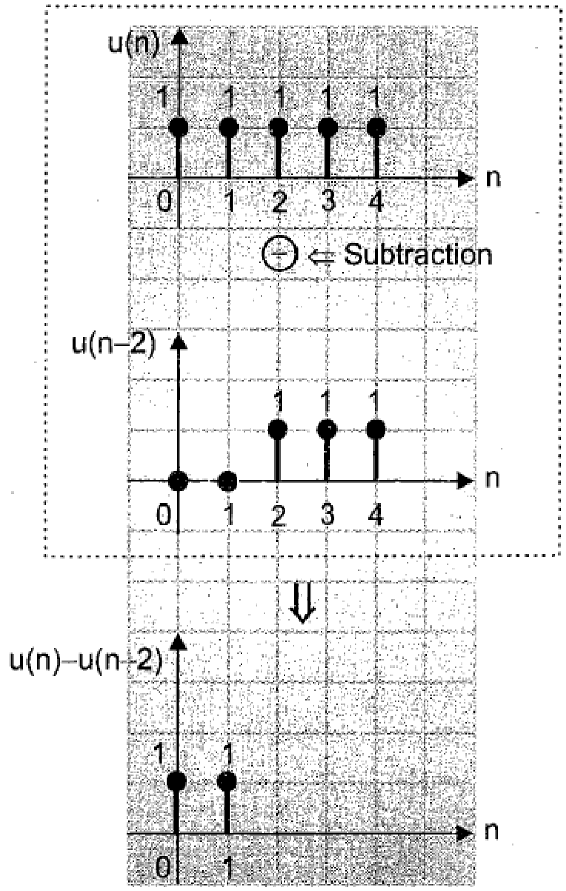


Fig. F-5 : $x(n) = u(n) - u(n-2)$

Note: Using the Twiddle factor makes the computation of DFT very easy using Matrices, Hence, using computers.

$$\therefore \quad x(n) = \{1, 1\} \qquad \text{...(1)}$$

Determination of 2-point DFT :

For 2-point DFT, N = 2

$$\text{We have, } W_N = e^{-j\frac{2\pi}{N}} \qquad \therefore \quad W_2 = e^{-j\frac{2\pi}{2}} = e^{-j\pi}$$

$$\therefore \quad W_2^{kn} = e^{-j\pi kn} \qquad \text{...(2)}$$

We know that 'n' is from 0 to N − 1. In this case, 'n' is from 0 to 1. Similarly, 'k' is from 0 to N − 1. In this case 'k' is from 0 to 1.

Now the matrix $W_N = W_2^{kn} = e^{-j\pi kn}$ can be written as,

$$W_2^{kn} = \begin{array}{c} \\ k=0 \\ k=1 \end{array} \begin{array}{c} \begin{array}{cc} n=0 & n=1 \end{array} \\ \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix} \end{array} \qquad \text{...(3)}$$

According to Equation (2) we have,

$$W_2^{kn} = e^{-j\pi kn}$$

$$\text{For } kn = 0 \Rightarrow W_2^0 = e^{-j\pi \times 0} = e^0 = 1$$

$$\text{For } kn = 1 \Rightarrow W_2^1 = e^{-j\pi \times 1} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

Putting these values in Equation (3),

$$W_2^{kn} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \text{...(4)}$$

Now given sequence x (n) = {1, 1}. In the matrix form it can be written as,

$$\therefore \quad x_N = x(n) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \text{...(5)}$$

Now DFT matrix is given by,

$$X_N = [W_N]\, x_N$$

$$\therefore \quad X_N = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} (1\times1)+(1\times1) \\ (1\times1)+(1\times-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Thus 2-point DFT is,

$$\boxed{X(k) = \{2, 0\}} \qquad \text{...(6)}$$

Magnitude plot :

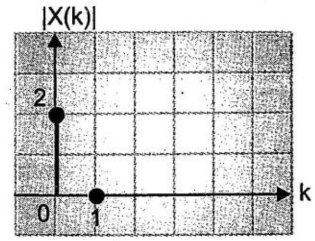


Fig. F-6 : Magnitude plot

We have, magnitude $= \sqrt{(\text{Real part})^2 + (\text{Imaginary part})^2}$

In Equation (6), the imaginary part is zero.

Thus magnitude at k = 0 is 2 and magnitude at k = 1 is 0. This magnitude plot is shown in Fig. F-6.

Determination of 4-point DFT :

For 4-point DFT, N = 4.

$$\text{We have,} \quad W_N = W_4 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$$

$$\therefore \quad W_N^{kn} = e^{-j\frac{\pi}{2}kn} \qquad \text{...(7)}$$

The range of K and n is from 0 to N − 1. That means 0 to 3.

$$[W_4] = W_4^{kn} = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array} \qquad \text{...(8)}$$

Now using Equation (7) we get,

$$W_4^0 = e^{-j\frac{\pi}{2}\times 0} = e^0 = 1$$

$$W_4^1 = e^{-j\frac{\pi}{2}\times 1} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$

$$W_4^2 = e^{-j\frac{\pi}{2}\times 2} = e^{-j\pi} = \cos\pi - j\sin\pi = -1$$

$$W_4^3 = e^{-j\frac{\pi}{2}\times 3} = e^{-j\frac{3\pi}{2}} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = +j$$

According to cyclic property of DFT.

$$W_4^0 = W_4^4 = 1$$

$$W_4^1 = W_4^5 = W_4^9 = -j$$

$$W_4^2 = W_4^6 = W_4^{10} = -1$$

and $$W_4^3 = W_4^7 = W_4^{11} = +j$$

Putting these values in Equation (8) we get,

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \quad ...(9)$$

Now given sequence is,

$x(n) = \{1, 1\}$. We want the length of this sequence equal to 4. It is obtained by adding zeros at the end of sequence. This is called as zero padding.

$$\therefore \quad x(n) = \{1, 1, 0, 0\}$$

$$\therefore \quad x_N = x_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad ...(10)$$

Now the DFT is given by, $X_N = [W_N]\, x_N$

$$\therefore \quad X_N = X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad X_N = X_4 = \begin{bmatrix} 1+1+0+0 \\ 1-j+0+0 \\ 1-1+0+0 \\ 1+j+0+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$\boxed{\therefore \quad X_4 = \{2, \quad 1-j, \quad 0, \quad 1+j\}}$$

This DFT sequence can also be written as,

$$X_4 = \{\underset{\substack{\uparrow \\ k=0}}{2+j0},\ 1-j,\ 0+j0,\ 1+j\}$$

Magnitude plot : The magnitude at different values is obtained as follows,

For $k = 0 \Rightarrow |X(k)| = \sqrt{(2)^2 + (0)^2} = 2$

For k = 1 $\Rightarrow |X(k)| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2} = 1.414$

For k = 2 $\Rightarrow |X(k)| = \sqrt{0+0} = 0$

For k = 3 $\Rightarrow |X(k)| = \sqrt{(1)^2 + (1)^2}$

$= \sqrt{2} = 1.414$

This magnitude plot is shown in Fig. F-7.

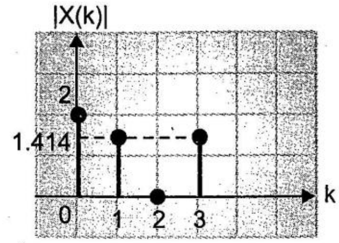


Fig. F-7 : Magnitude plot

Prob. 2 : Compute the DFT of four point sequence $x(n) = \{0, 1, 2, 3\}$

Soln. : The four point DFT in the matrix form is given by,

$$X_4 = [W_4] \cdot x(n)$$

$$\therefore \quad X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \quad X_4 = \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ 2j-2 \\ -2 \\ -2j-2 \end{bmatrix}$$

$$\boxed{\therefore \quad X_4 = \{\underset{\uparrow}{6}, 2j-2, -2, -2j-2\}}$$

Prob. 3 : Calculate 8 point DFT of :

$$x(n) = \{1, 2, 1, 2\}$$

Soln. : First we will make length of given sequence '8' by doing zero padding.

$$\therefore \quad x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\} \qquad ...(1)$$

We have, $\quad W_N = e^{-j\frac{2\pi}{N}}$

$$\therefore \quad W_8^{kn} = e^{-j\frac{2\pi}{8}} = e^{-j\frac{\pi}{4}kn} \qquad ...(2)$$

Here the range of K and n is from 0 to N – 1 that means from 0 to 7.

Now the matrix W_8^{kn} is as follows,

$$[W_8] = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \\ k=4 \\ k=5 \\ k=6 \\ k=7 \end{array} \begin{array}{c} \begin{array}{cccccccc} n=0 & n=1 & n=2 & n=3 & n=4 & n=5 & n=6 & n=7 \end{array} \\ \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \end{array} \quad \text{...(3)}$$

In Table F-1 we have already obtained different values of W_8^{kn}.

$$\therefore \quad W_8^0 = W_8^8 = W_8^{16} = W_8^{24} = W_8^{32} = W_8^{40} = ... = 1$$

$$W_8^1 = W_8^9 = W_8^{17} = W_8^{25} = W_8^{33} = W_8^{41} = W_8^{49} = ... = 0.707 - j\,0.707$$

$$W_8^2 = W_8^{10} = W_8^{18} = W_8^{26} = W_8^{34} = W_8^{42} = ... = -j$$

$$W_8^3 = W_8^{11} = W_8^{19} = W_8^{27} = W_8^{35} = W_8^{43} = ... = -0.707 - j\,0.707$$

$$W_8^4 = W_8^{12} = W_8^{20} = W_8^{28} = W_8^{36} = W_8^{44} = ... = -1$$

$$W_8^5 = W_8^{13} = W_8^{21} = W_8^{29} = W_8^{37} = W_8^{45} = ... = -0.707 + j\,0.707$$

$$W_8^6 = W_8^{14} = W_8^{22} = W_8^{30} = W_8^{38} = W_8^{46} = ... = j$$

$$W_8^7 = W_8^{15} = W_8^{23} = W_8^{31} = W_8^{39} = W_8^{47} = ... = 0.707 + j\,0.707$$

Now the DFT is given by,

$$X_8 = [W_8]\,x_n \quad \text{...(4)}$$

Putting values of W_8^{kn} in Equation (3) and write x_n in matrix form we get,

$$X_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707 - j\,0.707 & -j & -0.707 - j\,0.707 & -1 & -0.707 + j\,0.707 & j & 0.707 + j\,0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707 - j\,0.707 & j & 0.707 - j\,0.707 & -1 & 0.707 + j\,0.707 & -j & -0.707 + j\,0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707 + j\,0.707 & -j & 0.707 + j\,0.707 & -1 & 0.707 - j\,0.707 & j & -0.707 - j\,0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & 0.707 + j\,0.707 & j & -0.707 + j\,0.707 & -1 & -0.707 - j\,0.707 & -j & 0.707 - j\,0.707 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad X_8 = \begin{bmatrix} 1 + 2 + 1 + 2 + 0 + 0 + 0 + 0 \\ 1 + 1.414 - j\,1.414 - j - 1.414 - j\,1.414 + 0 + 0 + 0 + 0 \\ 1 - j\,2 - 1 + j\,2 + 0 + 0 + 0 + 0 \\ 1 - 1.414 - j\,1.414 + j + 1.414 - j\,1.414 + 0 + 0 + 0 + 0 \\ 1 - 2 + 1 - 2 + 0 + 0 + 0 + 0 \\ 1 - 1.414 + j\,1.414 - j + 1.414 + j\,1.414 + 0 + 0 + 0 + 0 \\ 1 + j\,2 - 1 - j\,2 + 0 + 0 + 0 + 0 \\ 1 + 1.414 + j\,1.414 + j - 1.414 + j\,1.414 + 0 + 0 + 0 + 0 \end{bmatrix}$$

$$\therefore \quad X_8 = \begin{bmatrix} 6 \\ 1 - j\,2.414 \\ 0 \\ 1 - j\,1.828 \\ -2 \\ 1 + j\,1.828 \\ 0 \\ 1 + j\,3.828 \end{bmatrix}$$

This is the required DFT.

Prob. 4 : Determine the length-4 sequence from its DFT.

$$X(k) = \{4,\ 1-j,\ -2,\ 1+j\}$$

Soln. : The IDFT in matrix form is given by,

$$\boxed{\text{IDFT} = x(n) = x_N = \frac{1}{N}\left[W_N^*\right] \cdot X_N} \qquad \text{...(1)}$$

Here X_N is the given DFT matrix. '*' indicates complex conjugate. To obtain the complex conjugate, we have to change the sign of j term. For example, complex conjugate of $1 - j1$ is $1 + j1$.

Now we have already obtained the matrix W_4 in problem (1). It is,

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \qquad \text{...(2)}$$

$$\therefore \quad [W_4^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \qquad \text{...(3)}$$

Given matrix of DFT is,

$$X_N = X_4 = \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix} \qquad \text{...(4)}$$

Putting Equations (3) and (4), and putting N = 4 in Equation (1) we get,

$$x_N = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$\therefore \quad x_N = \frac{1}{4}\begin{bmatrix} 4+1-j-2+1+j \\ 4+j-j^2+2-j-j^2 \\ 4-1+j-2-1-j \\ 4-j+j^2+2+j+j^2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 4+2+1+1 \\ 4-4 \\ 4+2-2 \end{bmatrix} \qquad \text{....as } j^2 = -1$$

$$\therefore \quad x_N = \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\therefore\ x(n) = \{1,\ 2,\ 0,\ 1\}}$$

Prob. 5 : Calculate the DFT of a sequence x (n) = {1, 1, 0, 0} and check the validity of your answer by calculating its IDFT.

Soln. : We will compute 4 point DFT. We have already obtained the matrix for $[W_4]$. It is,

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix}$$

Now given sequence is x (n) = {1, 1, 0, 0}

The DFT of this sequence is calculated in problem (1). It is $X_N = X(k) = \{2, 1-j, 0, 1+j\}$

Now we will check this answer by using the formulas for IDFT.

The IDFT is given by,

$$x(n) = \frac{1}{N}\left[W_N^*\right] \cdot X_N$$

$$\text{Here } \left[W_N^*\right] = \left[W_4^*\right] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\text{and } X_N = X_4 = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$\therefore \quad x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$\therefore \quad x(n) = \frac{1}{4}\begin{bmatrix} 2+1-j+0+1+j \\ 2+j+1+0-j+1 \\ 2-1+j+0-1-j \\ 2-j-1+0+j-1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad x(n) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

That means x (n) = {1, 1, 0, 0}

This is same as the given sequence; so calculated DFT is correct.

Prob. 6 : If $y(n) = \frac{[x(n)+x(-n)]}{2}$

Find Y (k) if $X(k) = \{0.5, 2+j, 3+j2, j, 3, -j, 3-j2, 2-j\}$

Soln. : We have,

$$y(n) = \frac{[x(n)+x(-n)]}{2} \qquad \text{...(1)}$$

Taking DFT of both sides we get,

$$Y(k) = \frac{[X(k)+X(-k)]}{2} \quad ...(2)$$

$$\text{Given} \quad X(k) = \{0.5, 2+j, 3+j2, j, 3, -j, 3-j2, 2-j\}$$

$$\therefore \quad X(-k) = \{0.5, 2-j, 3-j2, -j, +3, j, 3+j2, 2+j\}$$

Putting these values in Equation (2) we get,

$$Y(k) = \frac{1}{2}\{1, 4, 6, 0, 6, 0, 6, 4\}$$

$$\boxed{\therefore \; Y(k) = \{0.5, 2, 3, 0, 6, 0, 3, 2\}}$$

Properties of DFT :

In this section we will study some important properties of DFT. We know that, the DFT of discrete time sequence, $x(n)$ is denoted by $X(k)$. And the DFT and IDFT pair is denoted by,

$$x(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X(k)$$

Here 'N' indicates 'N' point DFT.

Linearity :

Statement : If $x_1(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_1(k)$ and $x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_2(k)$ then,

$$a_1 x_1(n) + a_2 x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} a_1 X_1(k) + a_2 X_2(k)$$

Periodicity :

Statement : If $x(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X(k)$ then

$$x(n+N) = x(n) \quad \text{for all } n$$

$$\text{and} \quad X(k+N) = X(k) \quad \text{for all } k.$$

Circular Convolution :

Statement : The multiplication of two DFTs is equivalent to the circular convolution of their sequences in time domain.

Mathematical equation :

$$\text{If } x_1(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} X_1(k) \text{ and } x_2(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} X_2(k) \text{ then,}$$

$$x_1(n) \, (N) \, x_2(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} X_1(k) \cdot X_2(k) \qquad ...(1)$$

Here (N) indicates circular convolution.

Prob. 1 : Given the two sequence of length 4 are :

$$x(n) = \{0, 1, 2, 3\}$$
$$h(n) = \{2, 1, 1, 2\}$$

Find the circular convolution.

Sol: here the given sequences are of length 4 so N = 4, so the circle will have only 4 points as shown:

Soln. : According to the definition of circular convolution,

$$y(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2((m-n))_N$$

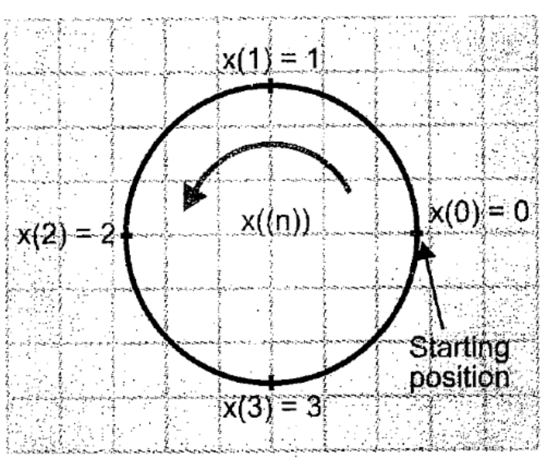


Fig. F-11 (a) : x (n) = {0, 1, 2, 3}

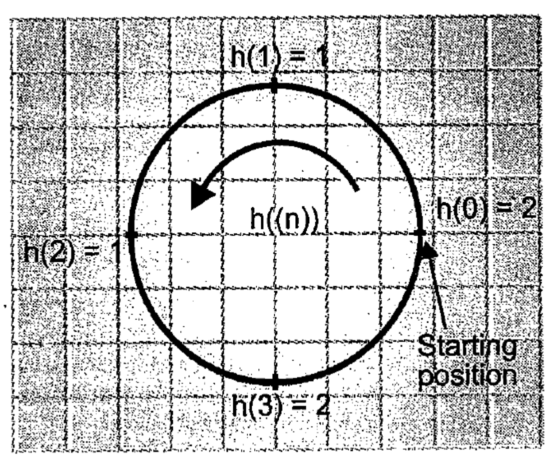
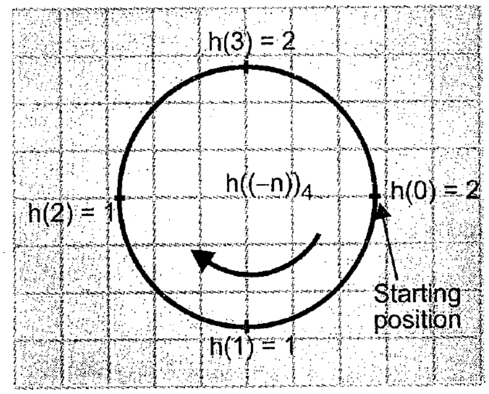


Fig. F-11 (b) : h (n) = {2, 1, 1, 2}

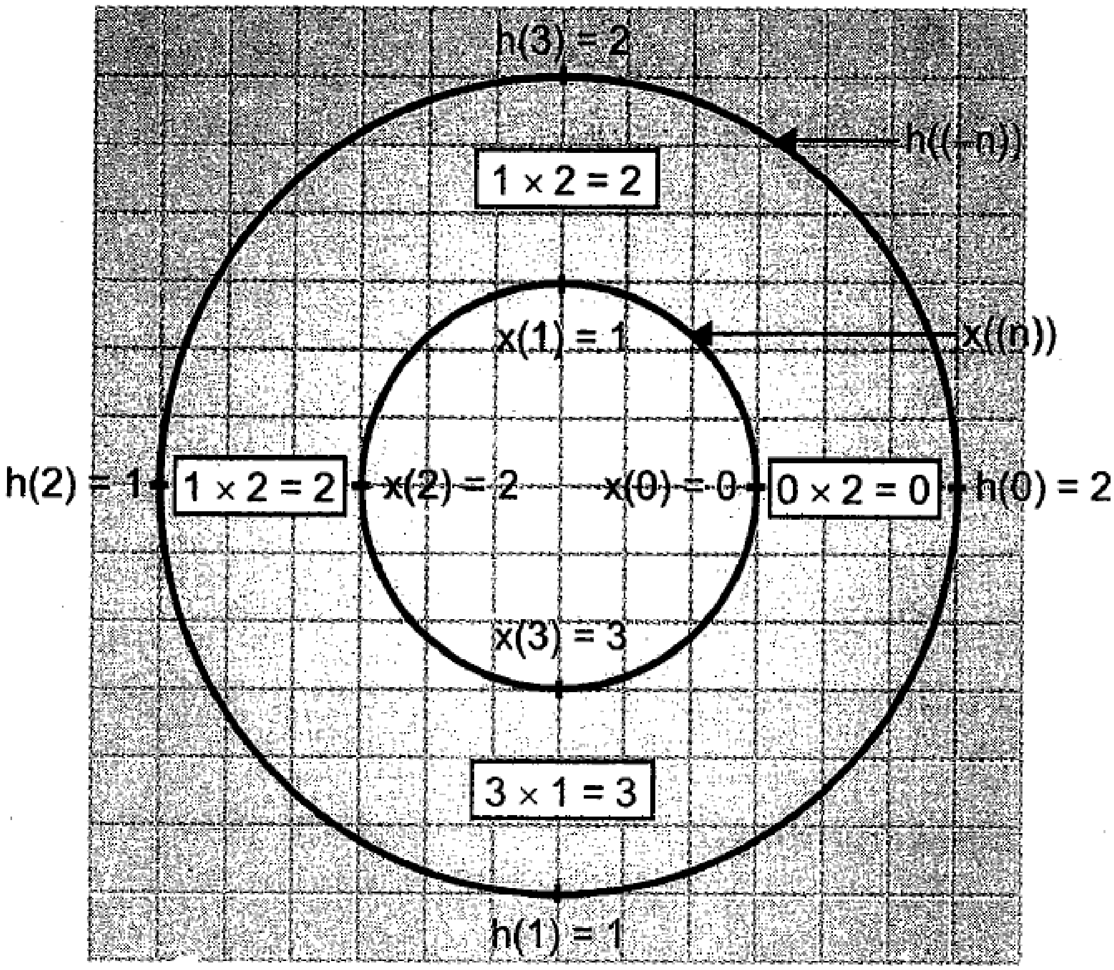
Step 1 – Finding y (0) here m = 0 makes the equation like this:



$$y(0) = \sum_{n=0}^{3} x(n)\,h((-n))_4$$

$$\therefore \quad y(0) = (0\times2)+(1\times2)+(1\times2)+(3\times1) = 0 + 2 + 2 + 3$$

$$\boxed{\therefore \ y(0) = 7}$$

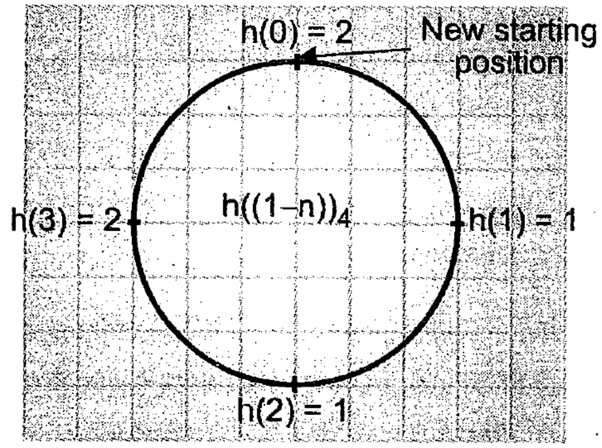


$$\sum_{n=0}^{3} \mathbf{x(n)\,h((-n))_4}$$

Step 2 –

Calculation of y (1) : Putting m = 1 in

$$y(1) = \sum_{n=0}^{3} x(n)\,h((1-n))_4$$



: $h((-n+1))_4$

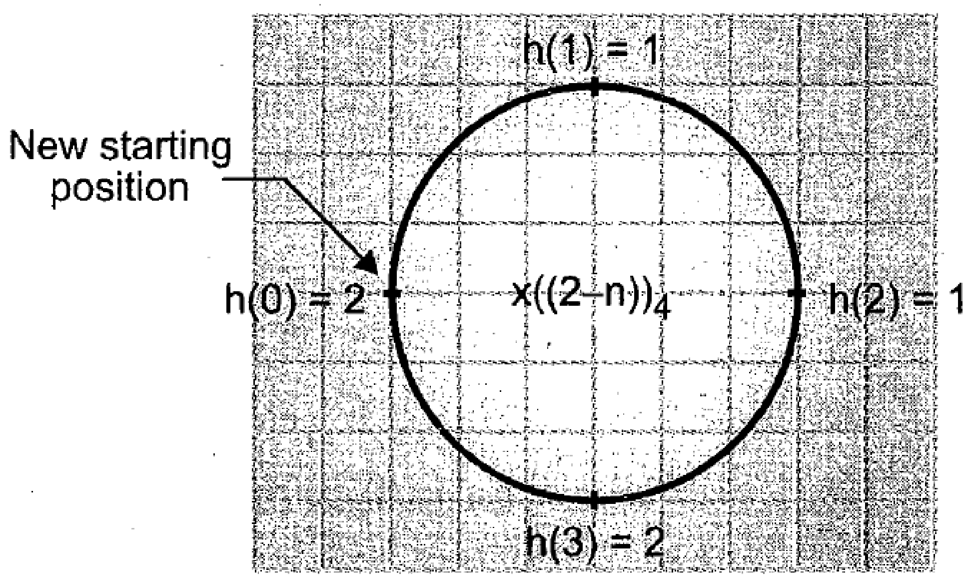
$$y(1) = (0\times1)+(3\times1)+(2\times2)+(1\times2) = 0+3+4+2$$

$$\therefore\ y(1) = 9$$

Step 3:

Calculation of y (2) : Putting m = 2 in

$$y(2) = \sum_{n=0}^{3} x(n)\,h((2-n))_4$$



: $h((-n+2))_4$

$$y(2) = (0\times1)+(3\times2)+(2\times2)+(1\times1) = 0+6+4+1$$

$$\therefore\ y(2) = 11$$

Step 3:

Calculation of y (3) :

Putting m = 3 in Equation

$$y(3) = \sum_{n=0}^{3} x(n)\, h((3-n))_4$$

$y(3) = (0\times2)+(3\times2)+(2\times1)+(1\times1) = 0+6+2+1$

$$\boxed{\therefore\ y(3) = 9}$$

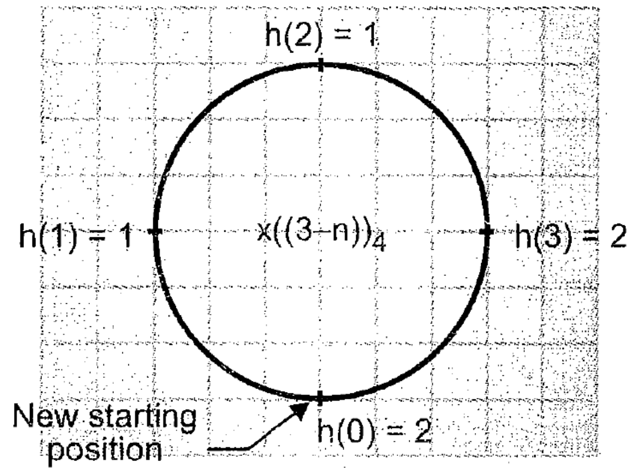


Fig. F-11(i) : $h((-n+3))_4$

Now the resultant sequence y (m) can be written as,

$$y(m) = \{y(0), y(1), y(2), y(3)\}$$

$$\boxed{\therefore\ y(m) = \{7, 9, 11, 9\}}$$

Prob. 2 : Using graphical method, obtain a 5-point circular convolution of two DT signals defined as,

$$x(n) = (1.5)^n, \qquad 0 \le n \le 2$$
$$y(n) = 2n-3, \qquad 0 \le n \le 3$$

Does the circular convolution obtained is same to that of linear convolution ?

Soln. : First we will obtain the sequences x (n) and y (n) by putting values of n as follows :

Given, $\quad x(n) = (1.5)^n, \quad 0 \le n \le 2$

For n = 0 $\Rightarrow x(0) = (1.5)^0 = 1$

For n = 1 $\Rightarrow x(1) = (1.5)^1 = 1.5$

For n = 2 $\Rightarrow x(2) = (1.5)^2 = 2.25$

$\therefore\ x(n) = \{1, 1.5, 2.25\}$ ...(1)

Now $y(n) = 2n-3, 0 \le n \le 3$

For n = 0 $\Rightarrow y(0) = 0-3 = -3$

For n = 1 $\Rightarrow y(1) = 2-3 = -1$

For n = 2 $\Rightarrow y(2) = 4-3 = 1$

For n = 3 $\Rightarrow y(3) = 6-3 = 3$

$$\therefore \quad y(n) = \{-3, -1, 1, 3\} \qquad \text{...(2)}$$

It is asked to calculate 5-point DFT. That means length of each sequence should be 5. This length is adjusted by adding zeros at the end of each sequence as follows (zero padding) :

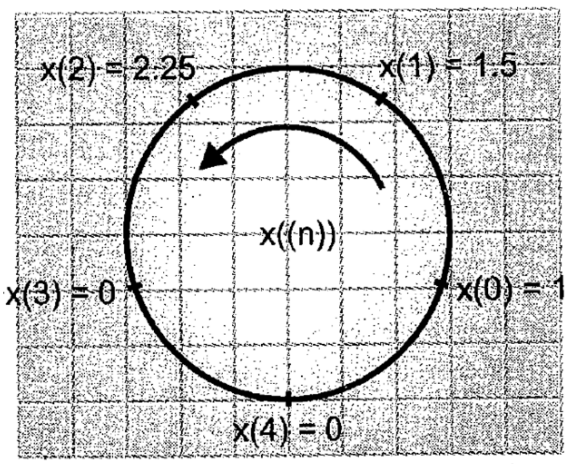
$$x(n) = \{1, 1.5, 2.25, 0, 0\} \qquad \text{...(3)}$$

$$\text{and} \quad y(n) = \{-3, -1, 1, 3, 0\} \qquad \text{...(4)}$$

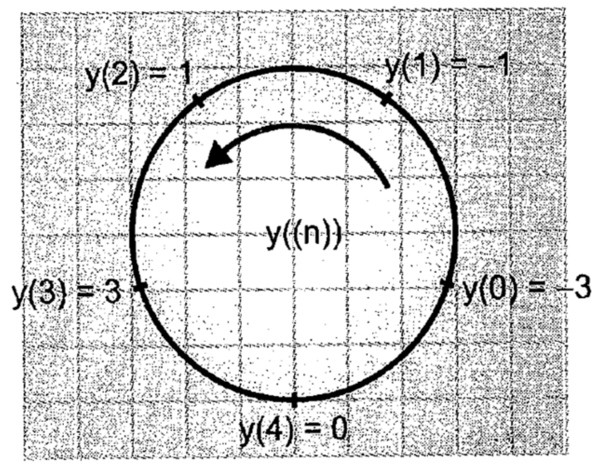
Now according to the definition of circular convolution we have,

Here the given sequences are x (n) and y (n) and length N = 5.

$$\therefore \quad y(m) = \sum_{n=0}^{4} x(n)\, y((m-n))_5$$

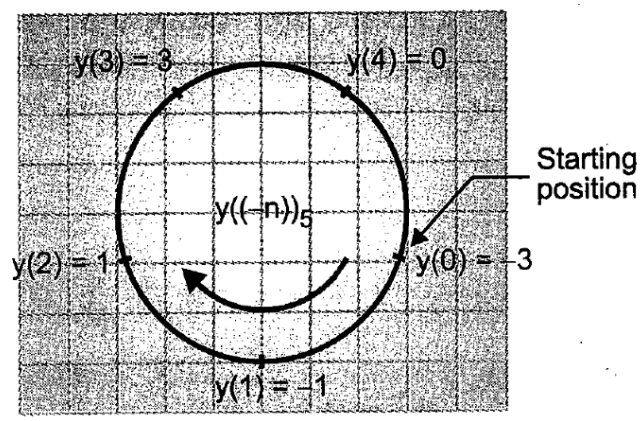


(a) **x (n) = {1, 1.5, 2.25, 0, 0}**



(b) **y (n) = {– 3, – 1, 1, 3, 0}**

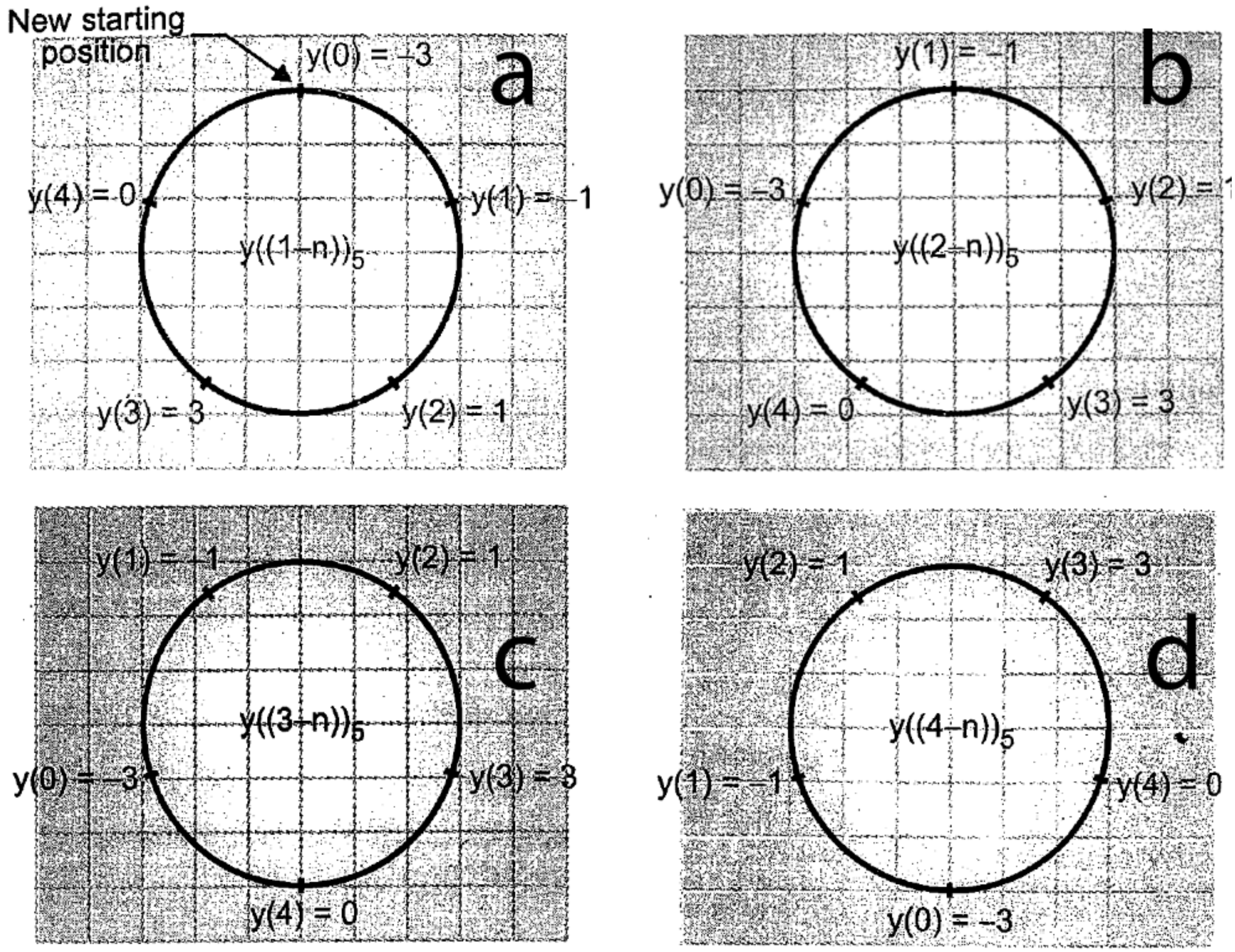
$$y(0) = \sum_{n=0}^{4} x(n)\, y((-n))_5$$



: y ((– n))

$$\therefore \quad y(0) = [1\times(-3)] + [0\times(-1)] + [0\times 1] + [2.25\times 3] + [1.5\times 0]$$
$$= -3 + 0 + 0 + 6.75 + 0$$

$$\boxed{\therefore\ y(0) = 3.75}$$



$y(1) = [1 \times (-1)] + [0 \times 1] + [0 \times 3] + [0 \times 2.25] + [1.5 \times (-3)] = -1 + 0 + 0 + 0 - 4.5$

$$\boxed{\therefore\ y(1) = -5.5}$$

$y(2) = [1 \times 1] + [0 \times 3] + [0 \times 0] + [2.25 \times (-3)] + [1.5 \times (-1)] = 1 + 0 + 0 - 6.75 - 1.5$

$$\boxed{\therefore\ y(2) = -7.25}$$

$$y(3) = [1 \times 3] + [0 \times 0] + [0 \times (-3)] + [2.25 \times (-1)] + [1.5 \times 1] \qquad \boxed{\therefore\ y(3) = 2.25}$$
$$= 3 + 0 + 0 - 2.25 + 1.5$$

$$y(4) = [1 \times 0] + [0 \times (-3)] + [0 \times (-1)] + [2.25 \times 1] + [1.5 \times 3] \qquad \boxed{\therefore\ y(4) = 6.75}$$
$$= 0 + 0 + 0 + 2.25 + 4.5$$

$y(m) = \{3.75, -5.5, -7.25, 2.25, 6.75\}$

Comparison with linear convolution :

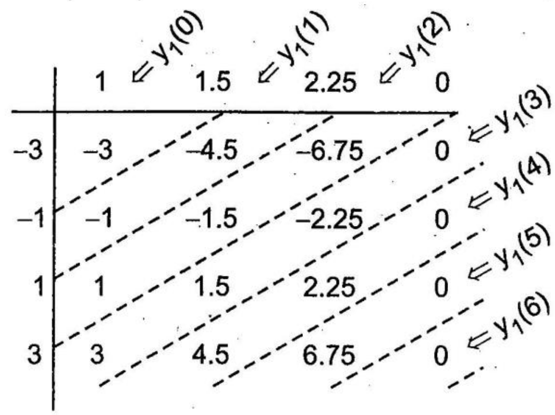
We will obtain linear convolution of two sequences using tabular method. We have,

$$x(n) = \{1,\ 1.5,\ 2.25\} = \{1,\ 1.5,\ 2.25,\ 0\}$$

and $$y(n) = \{-3,\ -1,\ 1,\ 3\}$$

Let $$y_1(n) = x(n) * y(n)$$

The linear convolution of x (n) and y (n) is shown in Fig. F-12(m).



x (n) * y (n)

From Fig.

$$y_1(0) = -3$$

$$y_1(1) = -1 - 4.5 = -5.5$$

$$y_1(2) = 1 - 1.5 - 6.75 = -7.25$$

$$y_1(3) = 3 + 1.5 - 2.25 = 2.25$$

$$y_1(4) = 4.5 + 2.25 = 6.75$$

$$y_1(5) = 6.75 + 0 = 6.75$$

$$y_1(6) = 0$$

Thus $$x(n) * y(n) = \{-3, -5.5, -7.25, 2.25, 6.75, 6.75, 0\}$$

Equations show that circular convolution and linear convolution are not same.

Prob. 4 : Use the four point DFT and IDFT to determine the circular convolution of sequences

$$x_1(n) = (\underset{\uparrow}{1}, 2, 3, 1)$$

$$x_2(n) = (\underset{\uparrow}{4}, 3, 2, 2)$$

Soln. : The four point DFT of $x_1(n)$ is $X_1(k)$ and it is given by,

$$X_1(k) = [W_4]\, x_{1N}$$

We have,
$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\therefore \quad X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore \quad X_1(k) = \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$\therefore \quad X_1(k) = \{7, -2-j, 1, -2+j\} \qquad \text{...(1)}$$

Similarly,
$$X_2(k) = [W_4]\, x_{2N}$$

$$\therefore \quad X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$$\therefore \quad X_2(k) = \begin{bmatrix} 4+3+2+2 \\ 4-3j-2+2j \\ 4-3+2-2 \\ 4+3j-2-2j \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$\therefore \quad X_2(k) = \{11, 2-j, 1, 2+j\}$$

Now according to property of circular convolution,

$$x_1(n) \; N \; x_2(n) \longrightarrow X_1(k) \cdot X_2(k) = X_3(k)$$

$$\therefore \quad X_3(k) = \{7, -2-j, 1, -2+j\} \cdot \{11, +2-j, 1, 2+j\}$$

$$\therefore \quad X_3(k) = \{77, -5, 1, -5\}$$

Let the result of $x_1(n)$ $\textcircled{N}$ $x_2(n)$ be sequence $x_3(n)$. It is obtained by computing IDFT of $X_3(k)$. According to the definition of IDFT we have,

$$x_3(n) = \frac{1}{N}[W_N^*] \cdot X_N$$

$$\therefore \quad x_3(n) = \frac{1}{4}[W_4^*] \cdot X_{3N}$$

$$\therefore \quad x_3(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}\begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$\therefore \quad x_3(n) = \frac{1}{4}\begin{bmatrix} 77-5+1-5 \\ 77-5j-1+5j \\ 77+5+1+5 \\ 77+5j-1-5j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

Considering real part only, approximately sequence $x_3(n)$ can be written as,

$$\boxed{x_3(n) = \{17, 19, 22, 19\}}$$

Prob. 3 : Determine the sequence

$$y(n) = x(n)\ \textcircled{N}\ h(n)$$

where $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

and $h(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$

Soln. :

We have, $y(m) = x_2(n)\ \textcircled{N}\ x_1(n) = h(n)\ \textcircled{N}\ x(n)$

Using matrix method,

Here $x(0) = 1$, $x(1) = 2$, $x(3) = 3$, $x(3) = 1$

and $h(0) = 4$, $h(1) = 3$, $h(2) = 2$, $h(3) = 2$

Here $N = 4$

In the matrix form we have

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} h(0) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(3) \\ h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ \\ y(1) \\ \\ y(2) \\ \\ y(3) \end{bmatrix} = \begin{bmatrix} (4\times1)+(2\times2)+(2\times3)+(3\times1) \\ \\ (3\times1)+(4\times2)+(2\times3)+(2\times1) \\ \\ (2\times1)+(3\times2)+(4\times3)+(2\times1) \\ \\ (2\times1)+(2\times2)+(3\times3)+(4\times1) \end{bmatrix} = \begin{bmatrix} 4+4+6+3 \\ \\ 3+8+6+2 \\ \\ 2+6+12+2 \\ \\ 2+4+9+4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

$$\therefore \quad y(m) = x(n) \; N \; h(n) = \{17, 19, 22, 19\}$$

Prob. 6 : Compute the circular convolution of following sequences and compare the results with linear convolution.

$$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$$
$$\text{and } h(n) = \{0, 1, 2, 3, 4, 3, 2, 1\}$$

Soln. : We have,

$$y(m) = x(n) \; N \; h(n)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} 0-1-2-3-4+3+2+1 \\ 0+1-2-3-4-3+2+1 \\ 0+1+2-3-4-3-2+1 \\ 0+1+2+3-4-3-2-1 \\ 0+1+2+3+4-3-2-1 \\ 0-1+2+3+4+3-2-1 \\ 0-1-2+3+4+3+2-1 \\ 0-1-2-3+4+3+2+1 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \\ -8 \\ -4 \\ 4 \\ 8 \\ 8 \\ 4 \end{bmatrix}$$

$$\therefore \quad y(m) = x(n) \; Ⓝ \; h(n) = \{-4, -8, -8, -4, 4, 8, 8, 4\} \qquad ...(1)$$
$$\uparrow$$

Now we will obtain linear convolution of given sequences as shown in Fig. F-13.

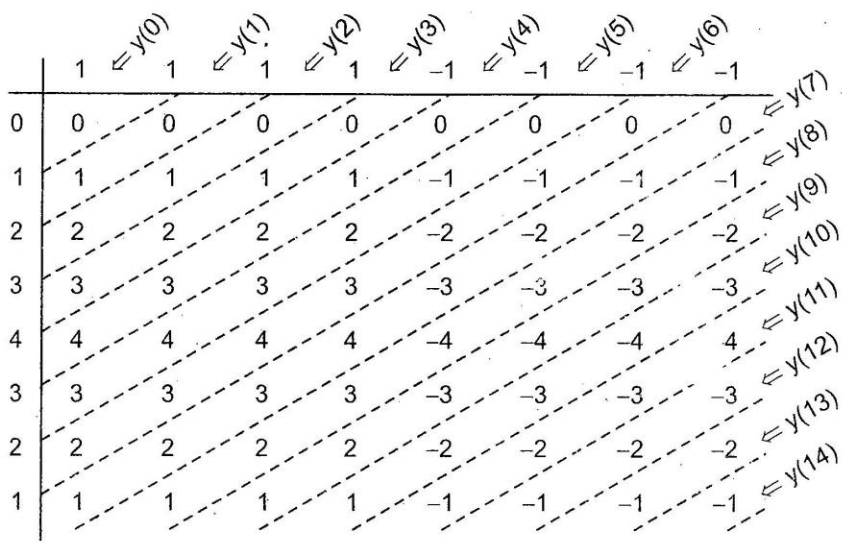


Fig. F-13 : x (n) ∗ h (n)

$$\therefore \quad y(n) = x(n) * h(n)$$

$$\therefore \quad y(n) = \{0, 1, 3, 6, 10, 11, 9, 4, -4, -9, -11, -10, -6, -3, -1\} \qquad ...(2)$$

From Equations (1) and (2) we can conclude that the results of circular convolution and linear convolution are not same.

More examples in Reference DSP Katre page 594